Magnetic Field Analysis with Nonconforming Voxel Modelling Using the Nested Geometric Multigrid Method

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Abstract — To establish a large scale magnetic field analysis, we have already proposed a magnetic field analysis using voxel modelling with nonconforming technique. The accuracy of flux densities and forces obtained from the nonconforming voxel modelling was verified in a 3D eddy current model as well as in a 2D nonlinear IPM motor model. In this paper, to make the proposed method more applicable large scale analysis method, the nested geometric multigrid method is introduced. The developed method is applied to a simple 2D model. It is shown that the convergence characteristic of magnetic field analysis with nonconforming mesh can be much improved by the multigrid method.

I. INTRODUCTION

Recently, a large scale magnetic field analysis can be carried out with the progress of computer technologies and numerical techniques, such as a parallel computing [1] and the multigrid method [2], [3], etc. However, it does not seem to have been established for an actual electrical machine with complex geometry for some reasons. For example, the CPU time for an automatic mesh generation increases as the number of elements increases. Moreover, in the multigrid method, the number of iterations increases drastically when the elements have distorted shape. To overcome these problems, the voxel modelling [4], in which the analyzed region is subdivided into square or cube elements uniformly, seems to be attractive because it provides us with easier mesh generation and better mesh quality without distortion. However, only few examples of the voxel modelling in magnetic field analysis, in which high accuracy is required, have been reported because error occurs due to rough shape approximation. If a large scale analysis is introduced to the voxel modelling, the error due to the shape approximation can be reduced because the size of elements can be small enough. Therefore, the large scale analysis using the voxel modelling seems attractive as the next generation of the magnetic field analysis. We have already proposed the method of magnetic field analysis using the voxel modelling with nonconforming technique [5], which can reduce the number of elements. Moreover, the effectiveness of the proposed method was shown. The accuracy of flux densities and forces obtained from the nonconforming voxel modelling was demonstrated in a 3D eddy current model as well as in a 2D nonlinear IPM motor model.

In this paper, to make the proposed method more applicable large scale magnetic field analysis, the nested geometric multigrid method is introduced. To investigate the convergence characteristics of magnetic field analysis using nonconforming voxel modelling, the developed method is applied to a simple 2D model.

II. METHOD OF ANALYSIS

In this section, the magnetic field analysis with the nonconforming voxel modelling using the multigrid method is illustrated using a simple 2D model.

A. Analysis Model

Fig. 1 shows a simple 2D linear magnetostatic analysis model of magnetic cylinders (relative permeability μ_r = 1000). Only a quarter region is analyzed due to symmetry. Two cylinders both with the radius of 10mm are placed on two sides of the y axis symmetrically with the distance of 30 mm between each other. When the uniform flux density B_{xo} = 1T is applied to the cylinders in x direction, the flux distribution is calculated.

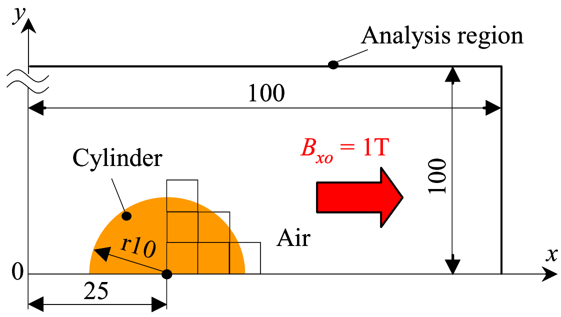

Fig. 1. A simple 2D model of cylinders (a quarter region).

B. Fundamental Equation

The 2D magnetic field analysis is carried out by using the 1st - order square finite element method with the magnetic vector potential $\boldsymbol{A}$. The fundamental equation is as follows:

$$\mathrm{rot}(\nu\,\mathrm{rot}\boldsymbol{A}) = 0 \qquad (1)$$

where ν is the reluctivity.

C. Voxel Modelling With Nonconforming Technique

In the voxel modelling, the analyzed region is subdivided into square elements uniformly. In this modelling, to reduce the outline approximation error of the analysis model, a huge number of elements is required. Therefore, to reduce the number of elements, the nonconforming technique [6] is applied to the voxel modelling. Fig. 2 shows the meshes of the nonconforming voxel modelling generated by the quadtree method [7], in which two rules are satisfied. One is that the regions including the geometric outlines are refined. The other is that the ratio of the large size to the small size in two neighboring elements is not larger than two. The number of unknown is 695 in the coarse mesh and it is 1,373 in the fine mesh.

In the analysis with nonconforming mesh, the potential on the nonconforming node at the center of the edge of the large element is interpolated linearly by the potentials on nodes at both side of the edge [6]. The flux distribution with the fine nonconforming voxel mesh is shown in Fig. 3. An appropriate result is obtained.

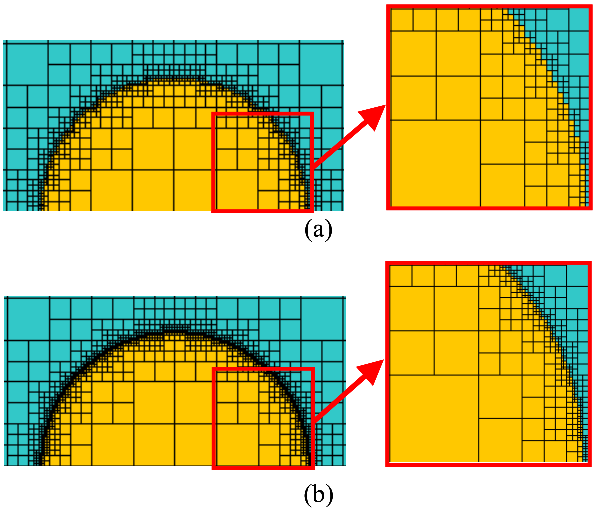

Fig. 2. Nonconforming voxel meshes of the magnetic cylinder, (a) coarse mesh, (b) fine mesh.

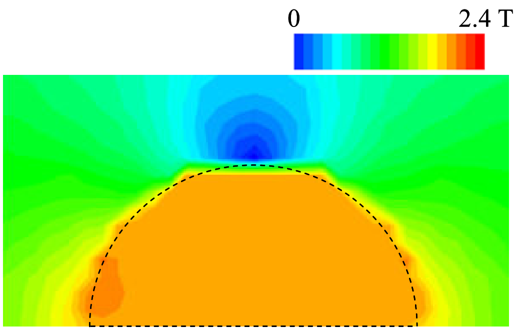

Fig. 3. Flux distribution.

D. *Multigrid Method*

The multigrid method, which is a numerical technique for large scale analysis, is introduced to the nonconforming voxel modelling. When the mesh generation technique mentioned above is used, the nested geometric multigrid method using the coarse and fine meshes, for example, shown in Fig. 2 (a) and (b) can be applied easily because every node in the coarse mesh is included in the nodes of the fine mesh. In this paper, the two-grid, such as the coarse and fine meshes show in Fig. 2, V-cycle of multigrid method is used. The Jacobi method is chosen for the solver of linear equations for both smoothing and residual equations because the analysis system can be easily parallelized in this case. The ordinary matrixes are used for the prolongation and restriction matrixes.

III. NUMERICAL RESULTS

The convergence characteristics of magnetic field analysis with the nonconforming voxel modelling using the multigrid method is investigated by using the double meshes shown in Fig. 2 (a) and (b). For reference, the analysis using the normal Jacobi method without the smoothing with the fine mesh only is also carried out. Fig. 4 shows the residual norm R versus number of iteration I. The residual norm of the multigrid method decreases rapidly compared with that of the normal Jacobi method. The total iterations until convergence with and without multigrid method are 5,219 and 19,298, respectively. Therefore, it is shown that the multigrid method is effective for the nonconforming voxel modelling.

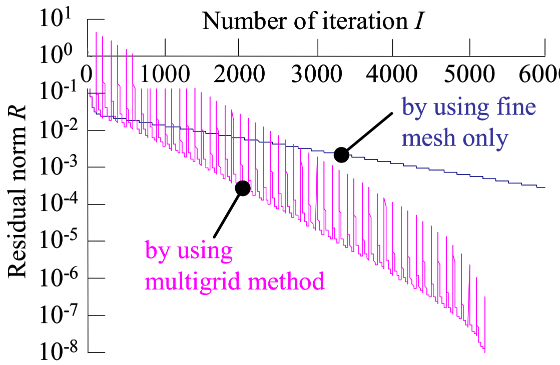

Fig. 4. Convergence characteristics of the multigrid method and the normal Jacobi method.

IV. CONCLUSION

The multigrid method is introduced to the magnetic field analysis using the nonconforming voxel modelling. It is shown that the multigrid method is effective for the nonconforming voxel modelling.

The investigation on the 3D analysis will be reported in the full paper.

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